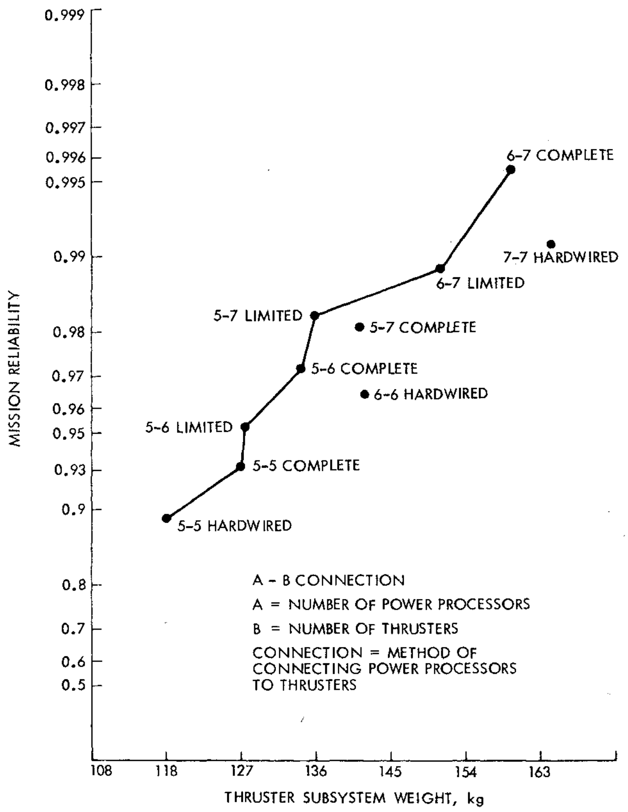

Fig. 1 Thrust subsystem weight vs mission reliability.

ple. But the point is that quite complicated definitions of success can be specified for a simulation. This is important because the definition of success has a very large impact on mission reliability and reliability tradeoff studies.

In the simulation, the mission was started with the nominal case, and as failures appeared the mission would continue as long as the degraded case could be performed. If the degraded case was achieved in the simulation, this was considered a mission success. The mission reliability (mission probability of success) was then obtained by taking the ratio of mission successes to the total number of missions simulated.

The actual performance of the simulation proceeded as follows. Failure times were generated for each component, based on the previously described failure distributions. Then, at the start of the simulation, the appropriate elements (thruster and power processors) were chosen and designated to be "active," so that operating time would be accumulated on these elements. The simulation then proceeded to the next "event," which would either be a failure or a phase change, (that is, a new scheduled thrusting requirement). For either event, new thrusters and processors would be selected and switch availability would be checked. In the event of failures, the scheduled "active" elements would not be available. In such cases, alternates were chosen, and switching and symmetry checked, until a viable configuration was found. In selecting "active" thrusters for a particular phase, the algorithms generally chose those with least accumulated active time to "even out" the operating times and to help prevent wearout. This procedure was then repeated until either the mission was complete, or until too many failures had occurred to continue the mission.

The other important factor in any reliability tradeoff study is weight. Figure 1 shows a means of presenting results that illustrates the dependence of reliability on weight. The solid line can be considered an "optimum" line in that configurations off this line are both heavier and less reliable.

Conclusions

The Monte Carlo simulations for the Encke Comet rendezvous mission have demonstrated the versatility of the Monte Carlo method for performing tradeoff studies. A great number of factors, usually too complicated to include, were used in the studies. Moreover, sensitivity studies indicated that most of the factors described above were pertinent to the results and so could not safely have been ignored. A general reliability simulation program is a very useful tool for analyzing competing designs from a reliability viewpoint.

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Model Reduction of a Nonlinear Rocket Control System

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Introduction

MODEL reduction by continued fraction is a very powerful and popular tool if it is used correctly.‡ In the original papers of Chen and Shieh, it has been pointed out that the system must be low pass in nature.[1,2] But in the limit-cycle analysis of a high order nonlinear rocket control system, one can not make sure whether the system is suitable for model reduction, although a Bode plot may help. It is the main purpose of this work to present such an application.

System Analysis

The block diagram of a flexible rocket control system is shown in Fig. 1. The linear blocks are[3]

$$TF_{\text{str.}} = [0.686(S+53)(S-53)(S^2-152.2S+14500)$$
$$\times (S^2+153.8S+14500)]/[(S^2+S+605)(S^2+45.5S+2660)$$
$$\times (S^2+2.51S+3900)(S^2+3.99S+22980)] \quad (1)$$

$$G_s(S) = 2750/(S^2+42.2S+2750) \quad (2)$$

$$G_r(S) = 7.21/(S+1.6)(S-1.48) \quad (3)$$

$$G_{sf}(S) = \frac{(S^2+70S+4000)(S^2+22S+12800)}{(S^2+30S+5810)(S^2+30S+12800)} \quad (4)$$

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‡One of the reviewers of this paper has pointed out that the method of continued fraction may sometimes reduce a stable system to an unstable one. This is a fact but will not be discussed here for lack of space. The authors are grateful to all the reviewers for their comments.

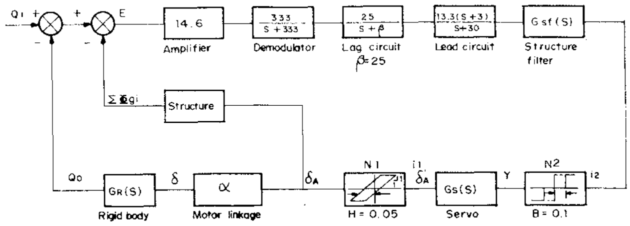

Fig. 1 Block diagram of a missile control system.

Using the method of model reduction, the reduced transfer functions of the structure block [Eq. (1)] can be obtained as[1,2]

$$TF_{5_{\text{str.}}} = \frac{-0.0244\,(S^2-128.9S+4144.4)\,(S^2+81.5S+1704.6)}{(S+41.3)\,(S^2+25.4S+2447.8)(S^2+0.97S+605.9)} \tag{5}$$

$$TF_{4_{\text{str.}}} = \frac{-0.015(S-99.8)(S^2-6.35S-2459.5)}{(S^2+0.91S+608.4)(S^2+22.7S+21.60.6)} \tag{6}$$

$$TF_{3_{\text{str.}}} = \frac{0.059\,(S+29.5)\,(S-37.2)}{(S+42.4)\,(S^2+1.59S+539.8)} \tag{7}$$

$$TF_{2_{\text{str.}}} = 0.0184(S-63.6)/(S^2+1.57S+417.7) \tag{8}$$

The structure loop of the considered system (without reduction) has been analyzed in Ref. 4 in which the frequency and amplitude of a limit cycle have been found. Here the system with different order of structure transfer functions is simulated on a digital computer. The obtained amplitudes and frequencies of limit cycles are tabulated in Table 1, where I_1 and I_2 are the amplitudes of the input sinusoids to the nonlinearities N_1 and N_2, respectively, and the asterisk represents the magnitude of the fundamental component of a limit cycle.

In Table 1, the simulated results of the original system (without reduction) have been checked with those obtained in Ref. 4. But it is evident that the second and third order models cannot be used for limit-cycle analysis since the errors are too large.

Now consider the overall system (with the structure loop and the rigid-body loop). In Fig. 1, assume that α and β are two adjustable parameters, and G_{d1} and G_{d2} are the describing functions of the nonlinearities N_1 and N_2, respectively, then the characteristic equation of the system is

$$(S+\beta)A(S) + B(S) + G_{d1}G_{d2} + 2G_{d1}G_{d2}C(S) = 0 \tag{9}$$

where

$$G_{d1} = \frac{1}{\pi}\left\{\frac{\pi}{2} + \sin^{-1}\left(1-\frac{0.05}{I_1}\right) + 2\left(1-\frac{0.05}{I_1}\right)\left[\frac{0.025}{I_1}\left(1-\frac{0.025}{I_1}\right)\right]^{1/2}\right\} - j\,\frac{0.1[1-(0.025/I_1)]}{\pi I_1} \tag{10}$$

Table 1 Limit-cycle values

Order of $TF_{\text{str.}}$	Frequency (rad/sec)	Amplitudes I_1	I_2
original	48.2	1.68	0.69
5th	48.6	1.65	0.62
3rd	14.6	1.02	0.24*
2nd	13.7	1.04*	0.31*

$$G_{d2} = \frac{4}{\pi I_2}\left(1-\frac{0.01}{I_2^2}\right)^{1/2}, \quad A(S) = \sum_{i=0}^{18} a_i S^i \tag{11}$$

$$B(S) = \sum_{i=0}^{13} b_i S^i, \quad C(S) = \sum_{i=0}^{13} c_i S^i \tag{12}$$

and

a_0	= −7.13881147E29§	a_{16}	= 1.27188781E05
a_1	= −2.00416664E28	a_{17}	= 5.18320000E02
a_2	= 3.00609734E29	a_{18}	= 1.00000000E00
a_3	= 2.375332070E28	b_0	= 6.55097808E29
a_4	= 1.56918609E27	b_1	= 1.97830887E29
a_5	= 7.41994753E25	b_2	= −2.84112521E29
a_6	= 2.58154582E24	b_3	= −9.77738076E28
a_7	= 7.43931665E22	b_4	= −1.79224149E27
a_8	= 1.64826083E21	b_5	= −1.56879222E24
a_9	= 3.1995779E19	b_6	= 4.47033368E23
a_{10}	= 4.84443778E17	b_7	= 9.74995237E21
a_{11}	= 6.63694244E15	b_8	= 6.68762499E19
a_{12}	= 6.90351694E13	b_9	= 9.27228544E17
a_{13}	= 6.71567502E11	b_{10}	= 4.12985263E15
a_{14}	= 4.60433051E09	b_{11}	= 6.57493654E13
a_{15}	= 3.10024268E08	b_{12}	= 2.94959125E11

b_{13}	=3.04961875E09
c_0	=7.26453543E32
c_1	=2.70322764E32
c_2	=1.16429355E31
c_3	=8.16488122E29
c_4	=2.39720815E28
c_5	=7.29484173E26
c_6	=1.30851682E25
c_7	=2.37865654E23
c_8	=2.67707162E21
c_9	=3.19247876E19
c_{10}	=2.02308369E17
c_{11}	=1.73862741E15
c_{12}	=4.74371307E12
c_{13}	=3.20521153E10

Substituting $S=j\omega$ into Eq. (9) and introducing the relation

$$I_1 = I_2 \left| 2750/[(j\omega)^2 + 42.2j\omega + 2750] \right| \tag{13}$$

into Eq. (10), the real part and the imaginary part of Eq. (9) can be written as

$$F_R(\alpha,\beta,I_2,\omega) = 0 \tag{14}$$

$$F_I(\alpha,\beta,I_2,\omega) = 0 \tag{15}$$

Solving these two equations for α and β, one has

$$\alpha = \alpha(I_2,\omega) \tag{16}$$

$$\beta = \beta(I_2,\omega) \tag{17}$$

which can be used to plot the loci of constant amplitude (I) and constant frequency (ω) in a parameter-plane as shown in Fig. 2.[5]

Assume $\alpha=0.3$ and $\beta=25$, then a point (Q_1) in Fig. 2 is defined, which indicates that the system has a limit cycle with amplitude and frequency equal to 1.03 and 16.2, respectively.

In the computer simulation, the system is first treated as that in Fig. 1, and then the structure transfer function is replaced by its low-order models. The results are given in Table 2.

§The numbers following E are the exponents to the base 10.

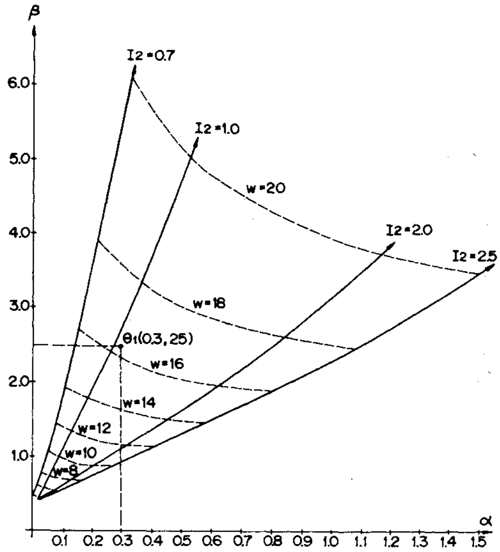

Fig. 2 Limit-cycle loci of a rocket control system.

Table 2 Limit-cycle values

Order of structure	Frequency (rad/sec)	Amplitudes I_1	I_2
original	15.7	1.34	1.04
5th	15.68	1.25	0.97
4th	17.4	1.32	0.54
3rd	15.75	1.33	1.05
2nd	15.65	1.26	0.97

Table 3 Limit-cycle values

Order of TF_{str}	Frequency (rad/sec)	Amplitudes I_1	I_2
original	64	1.20	1.32
5th	40	1.62	1.001
3rd	46.6	1.605	0.318
2nd	12.55	1.19*	0.32*

Table 4 Limit-cycle values

Order of TF_{str}	Frequency (rad/sec)	Amplitudes I_1	I_2
original	64	1.2	1.32
5th	13.55	1.31	1.67
4th	14.3	1.24	1.04
3rd	13.7	1.36	1.37
2nd	13.2	1.39	1.47

It can be seen that the structure transfer function can be replaced by its lower-order models without producing appreciable errors on limit cycles.

Control System with Low Damping Ratio

From Ref. 3, the original transfer function of the structure filter is

$$G_{sf}(S) = \frac{(S^2+12S+5810)(S^2+22S+12800)}{(S^2+13S+3520)(S^2+20.8S+21000)} \tag{18}$$

With this structure filter, the damping ratio of the structure loop is very low.[4] Using the same method as in the previous section, the simulated results for the structure loop are given in Table 3. The asterisk again represents the magnitude of the fundamental component of a limit cycle. For the overall system (with motor-linkage gain equal to 0.3), the simulated results are given in Table 4.

As the gain of the motor-linkage was changed to 0.1, a limit cycle was found at $\omega = 15$ and $I_2 = 4.34$. This result has been checked by computer simulations with both the original model and the reduced models. From these results, it can be seen that the analyses by use of the low-order models give correct results only if the limit-cycle frequency is low and the system damping is large.

Conclusions

For limit-cycle analysis of a nonlinear rocket control system, it has been shown that the reduced models can give good approximation to the original system only if the frequency of the limit cycle is low and the damping of the system is large. Since the frequency of a limit cycle can be found only after the analysis is completed, it is, therefore, advisable to use the original transfer functions instead of the reduced ones for limit-cycle analysis of the nonlinear rocket control system.

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Temperature Distribution in a Sublimation-Cooled Coated Cylinder in Convective and Radiative Environments

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Nomenclature

a	= thermal diffusivity coefficient
Bi	= Biot criteria
c	= heat capacity
Fo	= Fourier number
h	= heat transfer coefficient
ΔH	= enthalpy rate
$J_k(x)$	= Bessel function of first kind of order k on argument x.
K	= thermal conductivity coefficient
$\dot{m}$	= transpiration or sublimation rate
r	= radial coordinate
R	= Radius of the cylinder
Sk	= Stark number
$T(r,t)$	= temperature distribution
t	= time variable
x	= dimensionless radial coordinate

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